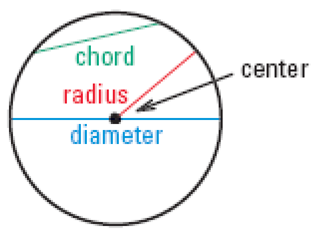
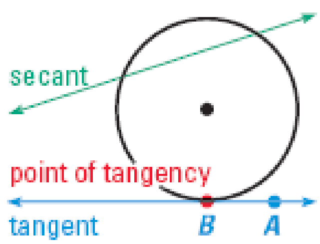
10.1 – Properties of Tangents

A **circle** is the set of all points in a plane equidistant from a given point called the **center** of the circle. A segment whose endpoints are the center and any point on the circle is a **radius**.

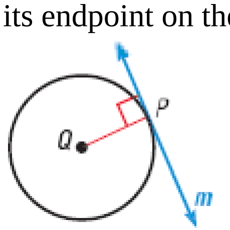
A **chord** is a segment whose endpoints are on a circle. A diameter is a chord that contains the center of the circle. It is also the longest chord and is equal to twice the length of the radius



A **secant** is a line that intersects a circle in two points. A **tangent** is a line, ray or segment in the plane of the circle that intersects a circle in exactly one point, the *point of tangency*.

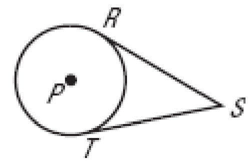


Thm 10.1: Perpendicular Tangent Thm
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.



Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

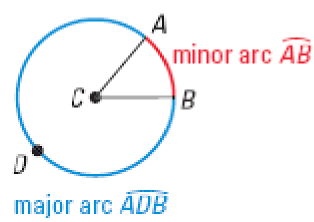
Thm 10.2: Common Tangents Thm
Tangent segments from a common external point are congruent.



If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

10.2 – Arc Measures

A **central angle** of a circle is angle whose vertex is the center of the circle. In the diagram below, angle ACB is a central angle of circle C.



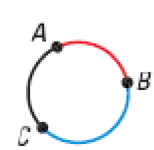
An **arc** is a portion of a circle that can be measured in degrees. The measure of an arc is equal to the measure of its central angle.

Arcs in the diagram above…

- $\overset{\frown}{AB}$ is a minor arc (less than 180º)
- $\overset{\frown}{ADB}$ is a major(more than 180º)

An arc that measures exactly 180º is called a **semicircle**.

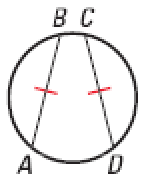
Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

10.3 – Properties of Chords

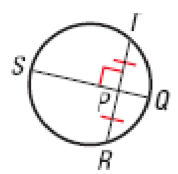
Thm 10.3: Congruent Chords Thm
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

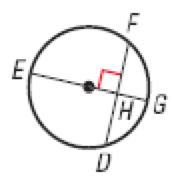
Thm 10.4: Perpendicular Chords Thm
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

In the diagram below, if $\overline{QS}$ is a $\perp$ bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

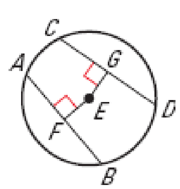


Thm 10.5: Perpendicular Bisector Thm
If a diameter is $\perp$ to a chord, then the diameter bisects the chord and its arc.

Below, $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.



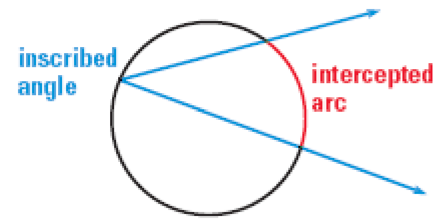
Thm 10.6: Equidistant Chords Thm
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



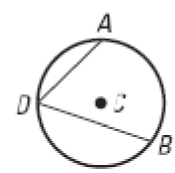
$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

10.4 – Inscribed Angles and Polygons

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

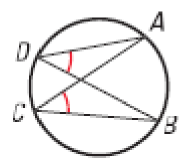


Thm 10.7: Inscribed Angle Thm
The measure of an inscribed angle is equal to one-half the measure of its intercepted arc.



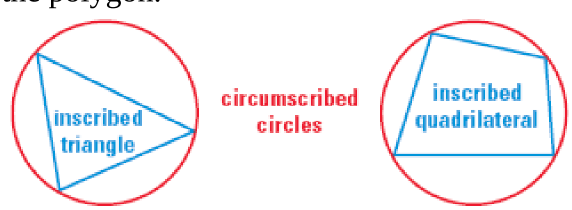
$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

Thm 10.8: Congruent Inscribed Angles
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

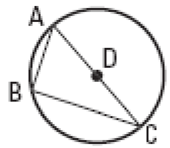


$\angle ADB \cong \angle ACB$

An **inscribed polygon** has all of its vertices on the circle. The circle is **circumscribed** on the polygon.



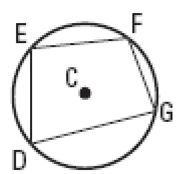
Thm 10.9: Inscribed Right Triangles
If a right triangles is inscribed in a circle, then the hypotenuse is a diameter.



$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

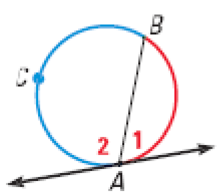
Thm 10.10: Inscribed Quadrilaterals
If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

In the diagram below, *D*, *E*, *F* and *G* lie on the circle. Thus $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$



10.5 – Other Angles in Circles

Thm 10.11: Tangent-Chord Angles
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



$m\angle 1 = \frac{1}{2} m\widehat{AB}$ $\quad m\angle 2 = \frac{1}{2} m\widehat{BCA}$

Thm 10.12: Angles Inside the Circle
If two chords intersect *inside* a circle, then the measure of each angle formed is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle (see diagram at top of next column).

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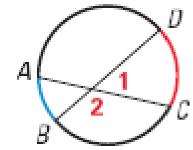
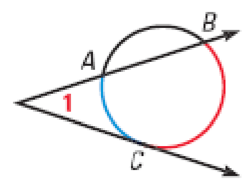


$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

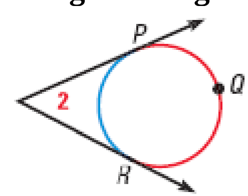
Thm 10.13: Angles Outside the Circle
If a tangent and a secant, two tangents or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.

Tangent-Secant



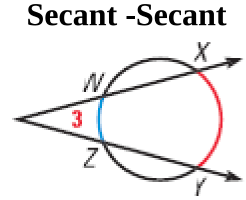
$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$

Tangent-Tangent



$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$

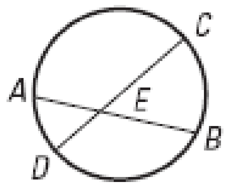
Secant -Secant



$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

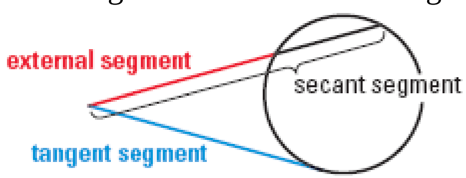
10.6 – Segment Lengths in Circles

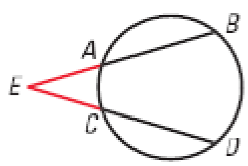
Thm 10.14: Segments of Chords Thm
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

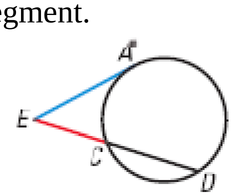
Thm 10.15: Segments of Secants Thm
If two secant segments share the same endpoint outside a circle, then the product of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.





$EA \cdot EB = EC \cdot ED$

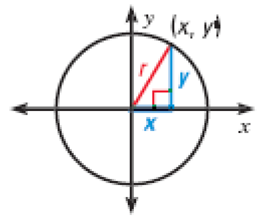
Thm 10.16: Segments of Secants/Tangents
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the tangent segment.



$EA^2 = EC \cdot ED$

10.7 – Equations and Graphs of Circles

Circles Centered at the Origin
Let (x, y) represent any point on a circle with center at the origin and radius *r*.

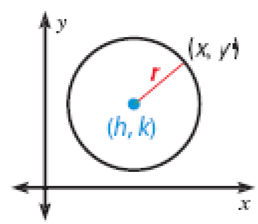


By the Pythagorean Theorem,

$$x^2 + y^2 = r^2$$

This is the equation of a circle with radius *r* and center at the origin.

Circles Centered at (h, k)
Suppose a circle has radius *r* and center (h, k). Let (x, y) be a point on the circle with center at the origin and radius *r*.



The distance between (x, y) and (h, k) is *r*, and by the Distance Formula

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

Square both sides to find the standard equation of a circle

Standard Equation of a Circle
The standard equation of a circle with center (h, k) and radius *r* is:

$$(x-h)^2 + (y-k)^2 = r^2$$