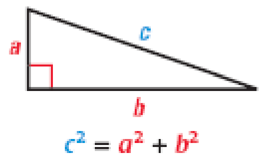
7.1 – The Pythagorean Theorem

The Pythagorean Theorem
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



Pythagorean Triples – A set of three integers *a*, *b* and *c* that satisfy the equation

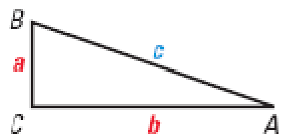
$$a^2 + b^2 = c^2$$

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250

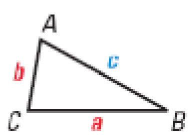
7.2 – Converse of Pythagorean Theorem

Converse of the Pythagorean Theorem
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangles is a right triangle.



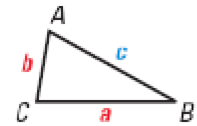
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is right.

Theorem 7.3
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangles is an acute triangle.



If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

Theorem 7.4
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangles is an obtuse triangle.



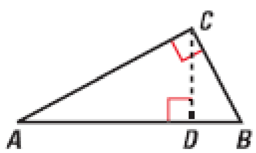
If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

Concept Summary

If…	Then…
$c^2 = a^2 + b^2$	Right Triangle
$c^2 < a^2 + b^2$	Acute Triangle
$c^2 > a^2 + b^2$	Obtuse Triangle

7.3 – Similar Right Triangles

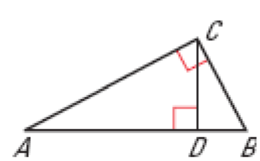
Theorem 7.5
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



So $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$ and $\triangle CBD \sim \triangle ACD$

Geometric Mean Altitude Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

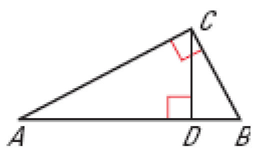


So,

$$\frac{AD}{CD} = \frac{CD}{BD}$$

Geometric Mean Legs Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



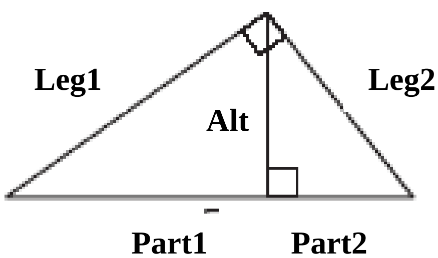
So,

$$\frac{AB}{CB} = \frac{CB}{DB}$$

and,

$$\frac{AB}{AC} = \frac{AC}{AD}$$

Concept Summary



Altitude Theorem

$$\frac{\text{Part1}}{\text{Alt}} = \frac{\text{Alt}}{\text{Part2}}$$

Legs Theorem

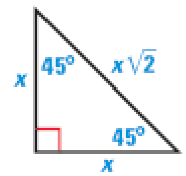
$$\frac{\text{Part1}}{\text{Leg1}} = \frac{\text{Leg1}}{\text{Hyp}}$$

and

$$\frac{\text{Part2}}{\text{Leg2}} = \frac{\text{Leg2}}{\text{Hyp}}$$

7.4 – Special Right Triangles

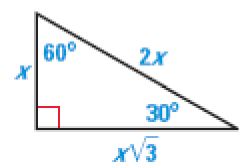
45°-45°-90° Triangle Theorem
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



$$\text{hypotenuse} = \text{leg} \bullet \sqrt{2}$$

$$\text{leg} = \frac{\text{hypotenuse}}{\sqrt{2}}$$

30°-60°-90° Triangle Theorem
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer is leg is $\sqrt{3}$ times as long as the shorter leg.



$$\text{hypotenuse} = 2 \bullet \text{shorter leg}$$

$$\text{longer leg} = \text{shorter leg} \bullet \sqrt{3}$$

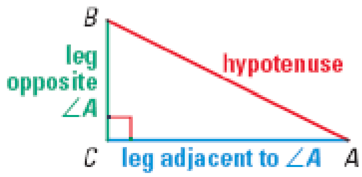
$$\text{shorter leg} = \frac{\text{hypotenuse}}{2}$$

$$\text{shorter leg} = \frac{\text{longer leg}}{\sqrt{3}}$$

7.5 – The Tangent Ratio

Tangent Ratio
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows (see diagram below):

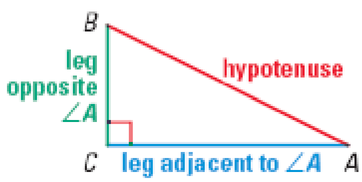
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



7.6 – The Sine and Cosine Ratios

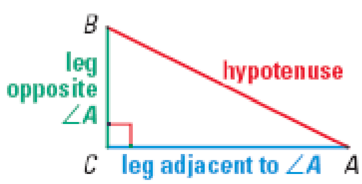
Sine Ratio
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ (written as sin A) is defined as follows (see diagram below):

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$



Cosine Ratio
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The cosine of $\angle A$ (written as cos A) is defined as follows (see diagram below):

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



Concept Summary
A commonly used device to help students remember the three trigonometric ratios is ***SOH-CAH-TOA***. This stands for

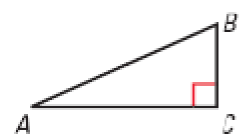
SOH	$\sin = \frac{opposite}{hypotenuse}$	*or*	$\sin = \frac{opp}{hyp}$
CAH	$\cos = \frac{adjacent}{hypotenuse}$	*or*	$\cos = \frac{adj}{hyp}$
TOA	$\tan = \frac{opposite}{adjacent}$	*or*	$\tan = \frac{opp}{adj}$

7.7 – Solving Right Triangles

Solve a Right Triangle – To solve a right triangle means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and one acute angle

Inverse Trigonometric Ratios
Let $\angle A$ be an acute angle.



Inverse Tangent
If tan $A = x$, then $\tan^{-1} = m\angle A$

Notation (diagram): $\tan^{-1}\left(\frac{BC}{AC}\right) = m\angle A$

Inverse Sine
If sin $A = x$, then $\sin^{-1} = m\angle A$

Notation (diagram): $\sin^{-1}\left(\frac{BC}{AB}\right) = m\angle A$

Inverse Cosine
If cos $A = x$, then $\cos^{-1} = m\angle A$

Notation (diagram): $\cos^{-1}\left(\frac{AC}{AB}\right) = m\angle A$