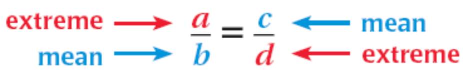
Geometry Notes – Chapter 6: Similarity

6.1 – Ratios, Proportion and Geo. Mean

Given two numbers *a* and *b* ($b \neq 0$), the **ratio of *a* to *b*** is $\frac{a}{b}$. The ratio of *a* to *b* can also be written as $a : b$.

A **proportion** is an equation that states that two ratios are equal.



The numbers *b* and *c* are the ***means*** of the proportion. The numbers *a* and *d* are the ***extremes*** of the proportion.

Property of Proportions

1. Cross Products – In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$,
then $ad = bc$.

Geometric Mean
The geometric mean of two positive numbers *a* and *b* is the positive number *x* that satisfies
$\frac{a}{x} = \frac{x}{b}$ so $x^2 = ab$ and $x = \sqrt{ab}$.

<u>For example</u>, the geometric mean of 4 and 8 is $\sqrt{4 \cdot 8} = \sqrt{32} = 4\sqrt{2} \approx 5.66$.

6.2 – Solving Problems with Proportions

Additional Properties of Proportions

2. *Reciprocal Property* – If two ratios are equal, their reciprocals are also equal.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}.$$

3. *Interchangeable Means* – If you interchange the means of a proportion, then you form another true proportion.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}.$$

4. *Denominator Addition* – In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

$$\text{If } \frac{a+b}{b} = \frac{c+d}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}.$$

6.3 – Similar Polygons

Two polygons are ***similar polygons*** if

1. Their corresponding angles are congruent
2. Their corresponding side lengths are proportional

For example, if $\triangle ABC \sim \triangle DEF$, then

$$\angle A \cong \angle D,\ \angle B \cong \angle E \text{ and } \angle C \cong \angle F$$

and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Note that the symbol for similarity is "**~**".

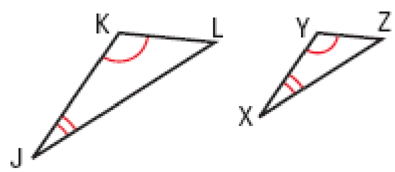
Scale Factor – If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the scale factor.

For example, for triangles ABC and DEF above, if the length of AB = 4 and DE = 10, then the scale factor of ABC to DEF is

$$\frac{4}{10} = \frac{2}{5}$$

6.4 – Prove Similarity with AA

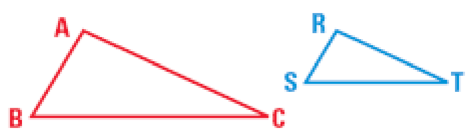
AA Similarity Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

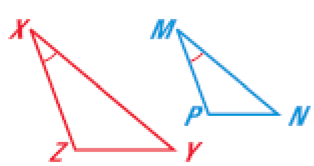
6.5 – Prove Similarity with SSS and SAS

SSS Similarity Theorem
If the corresponding side lengths of two triangles are proportional, then the two triangles are similar.



If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

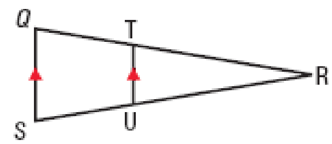
SAS Similarity Theorem
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the two triangles are similar.



If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

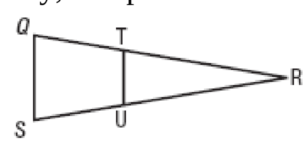
6.6 – Miscellaneous Proportionality Thms

Triangle Proportionality Theorem
If a line parallel to one side of a triangle intersects the other two sides, then it divides the other sides proportionally.



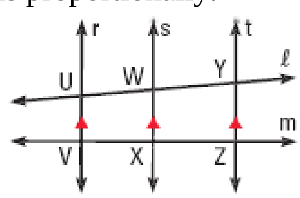
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Triangle Proportionality Converse
If a line divides two sides of a triangle proportionally, it is parallel to the third side.



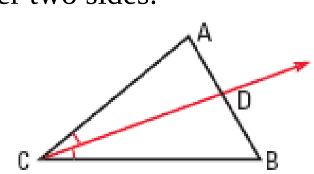
If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Triangle Proportionality Theorem
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$\frac{UW}{WY} = \frac{VX}{XZ}$

Triangle Proportionality Theorem
If a ray bisects an angle of a triangle, then it divides the opposite sides into segments whose lengths are proportional to the lengths of the other two sides.



$\frac{AD}{DB} = \frac{CA}{CB}$