Geometry Notes – Chapter 2: Reasoning and Proof

2.1 – Inductive Reasoning

Defined Terms

Conjecture – An unproven statement that is based on observations.
Inductive Reasoning – Finding a pattern in specific cases and then making a conjecture for the general case.
Counterexample – A specific case for which a conjecture is false.

Example #1

What is the next number in the sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, ___

Conjecture: after the first two 1's, each number appears to be the sum of the two previous numbers (2 = 1+1, 3 = 1+2, etc…)
Inductive Reasoning: Using our conjecture, we predict that the next number in the sequence is 21 + 34 = **55**.

Example #2

Conjecture: adding two numbers together always results in an even number
Counterexample: 3 + 4 = 7 (odd)

2.2 – Conditional Statements

A ***conditional statement*** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. This type of statement can be written in *if-then* form.

The **hypothesis** of the statement is the part following the **if** and the **conclusion** is the part following the **then**.

Example

The statement, *All dogs are mammals*
Can be written in if-then form, as follows:

If an animal is a dog, then it is a mammal.

Hypothesis: an animal is a dog
Conclusion: the animal is a mammal

Related Conditional Statements

To write the ***converse*** of a conditional statement, **exchange** the hypothesis and conclusion of the original statement.

Example

Original statement: $a \rightarrow b$

If it is raining, then I will get wet.

Converse statement: $b \rightarrow a$

If I am getting wet, then it is raining.

To write the ***inverse*** of a conditional statement, **negate** the hypothesis and conclusion of the original statement.

Example

Original statement: $a \rightarrow b$

If it is raining, then I will get wet.

Inverse statement: $\sim a \rightarrow \sim b$

If it is not raining, then I will not get wet.

To write the ***contrapositive*** of a conditional statement, **negate AND exchange** the hypothesis and conclusion of the original.

Example

Original statement: $a \rightarrow b$

If it is raining, then I will get wet.

Contrapositive statement: $\sim b \rightarrow \sim a$

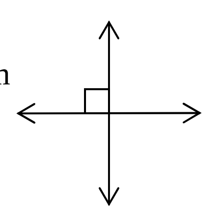
If I am not getting wet, then it is not raining.

Equivalent Statements

- The **contrapositive** always has an equivalent meaning to the **original**.
- The **converse** always has an equivalent meaning to the **inverse**.

Perpendicular Lines

If two lines intersect to form a right angle, then they are ***perpendicular lines***.



2.3 – Deductive Reasoning

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties and the laws of logic to form a logical argument – much like what you see in mystery movies or television shows such as *Sherlock Holmes* or *CSI*.

Laws of Logic

Law of Detachment
If the hypothesis of a true conditional statement is true, then the conclusion of the statement is also true.

Example
Assume the following to be true:

If it rains, then you will get wet.

Given – It is raining
Deduction – you will get wet.

Law of Syllogism

Form	Symbolization
If a, then b	$a \rightarrow b$
If b, then c	$b \rightarrow c$
$\therefore$ If a, then c	$\therefore a \rightarrow c$

If the first two statements are true, then the third statement must be true as well.

Example
Assume the following to be true:

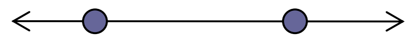
If it rains, then you will get wet.
If you get wet, then you will cough.

Then the following conclusion can be made by deductive logic:

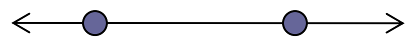
If it rains, then you will cough.

2.4 – Postulates and Diagrams

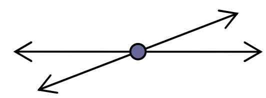
Postulate 5 – Through any two points there exists exactly on line.



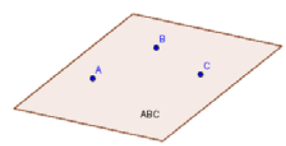
Postulate 6 – A line contains at least two points.



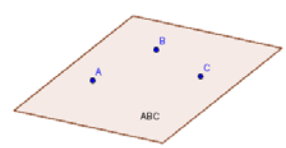
Postulate 7 – If two lines intersect, then their intersection is exactly one point.



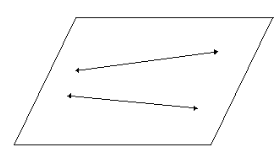
Postulate 8 – Through any three noncollinear points there exists exactly one plane.



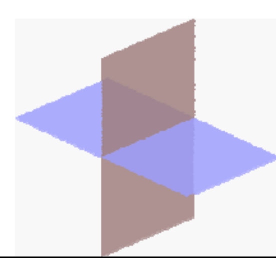
Postulate 9 – A plane contains at least three noncollinear points.



Postulate 10 – If two points lie in a plane, then the line containing them lies in the plane.



Postulate 11 – If two planes intersect, then their intersection is a line.



2.5 – Properties from Algebra

Algebraic Properties of Equality

Addition Property of Equality

If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality

If $a = b$, then $ac = bc$.

Division Property of Equality

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Substitution Property of Equality

If $a = b$, then you may replace b with a in any equation or expression (and vice-verse)

Distributive Property

$a(b + c) = ab + ac$

Reflexive Property of Equality

For any real number a, $a = a$

Symmetric Property of Equality

For any real numbers a and b, if $a = b$, then $b = a$

Transitive Property of Equality

For any real numbers a, b and c, if a = b *and* $b = c$, then $a = c$

2.6 – Segment and Angle Proofs

Congruence of Segments

Segment congruence is reflexive, symmetric and transitive.

Reflexive	$\overline{AB} \cong \overline{AB}$
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

Congruent of Angles

Angle congruence is reflexive, symmetric and transitive.

Reflexive	$\angle A \cong \angle A$
Symmetric	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$
Transitive	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$

2.7 – Angle Pair Relationships

Right Angles Congruence Theorem

All right angles are congruent

Congruent Supplements Theorem

If two angles are supplementary to the same angle, those angles are congruent.

Congruent Complements Theorem

If two angles are complementary to the same angle, those angles are congruent.

Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

Vertical Angles Congruence Theorem

Vertical angles are congruent.