Geometry Notes – Chapter 1: Essentials of Geometry

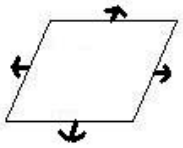
1.1 – Points, Lines and Planes

Undefined Terms

Point – A **point** has no dimension. It is represented by a dot. $\bullet A$

Line – A one-dimensional figure that extends without end in two directions.

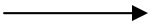
Plane – A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.



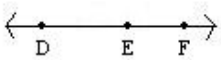
Defined Terms

Segment – Part of a line that consists of two endpoints and all the points between them.

Ray – A **ray** is part line, part segment. On one end it contains an endpoint; on the other it extends without end.



Opposite Rays – Two rays that have the same endpoint and extend in opposite directions, thus making a line together. Below, $\overrightarrow{ED}$ and $\overrightarrow{EF}$ are opposite rays.



Collinear – Two or more points that lie on the same line..

Coplanar – Three or more points that lie in the same plane.

Symbols

Object	Symbol
Line AB	$\overleftrightarrow{AB}$
Line Segment AB	$\overline{AB}$
Ray AB	$\overrightarrow{AB}$
Distance from A to B	AB

1.2 – Segments and Congruence

Ruler Postulate

Each point on a number line is assigned a unique number, called its **coordinate**.
The **distance** between two points is defined as the absolute value of the difference between of the coordinates of the points.

Segment Addition Postulate

If the point B is between points A and C, then the distance from A to B added to the distance from B to C is equal to the distance from A to C. So $AB + BC = AC$.

1.3 – Midpoint and Distance Formulas

Midpoint

The midpoint of a segment is the point that divides the segment into two congruent segments. So if M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

Midpoint Formula

The midpoint of two points with the coordinates (x_1, y_1) and (x_2, y_2) is found by the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance Formula

The distance between two points with the coordinates (x_1, y_1) and (x_2, y_2) is found by the formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

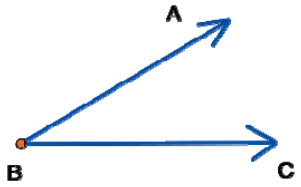
1.4 – Measure and Classify Angles

Angle

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle and the endpoint is the **vertex** of the angle.

Naming Angles

The **angle** below can be named 3 ways: $\angle ABC$, $\angle CBA$ or simply $\angle B$. The vertex must be the middle letter when using 3 points to name an angle.



Measuring Angles

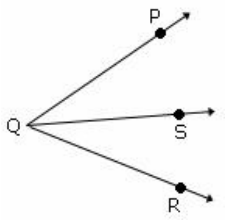
Angles are measured in degrees ($^\circ$). In Geometry, angles have measures greater than 0 degrees and up to 180 degrees.

Classifying Angles

Acute angle – has measure less than 90°.
Right angle – has measure of 90°.
Obtuse angle – has measure more than 90°, but less than 180°.
Straight angle – has measure of 180°.

Angle Addition Postulate

If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.



1.5 – Angle Pair Relationships

Complementary Angles

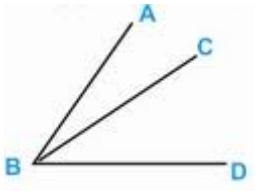
Two angles whose sum is 90 degrees. Each angle is the *complement* of the other.

Supplementary Angles

Two angles whose sum is 180 degrees. Each angle is the *supplement* of the other.

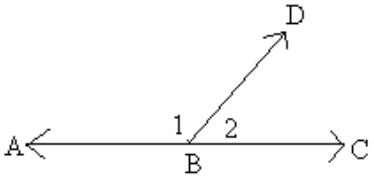
Adjacent Angles

Two angles that share a common vertex and side, but have no common interior points.



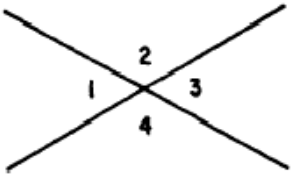
Linear Pair

Two adjacent angles form a ***linear pair*** if their noncommon sides are opposite rays (and thus, form a line). Angles in a linear pair are also supplementary, since a line makes 180°.



Vertical Angles

Two intersecting lines form two pairs of vertical angles. In the diagram below, angles 1 and 3 are vertical, as are angles 2 and 4.



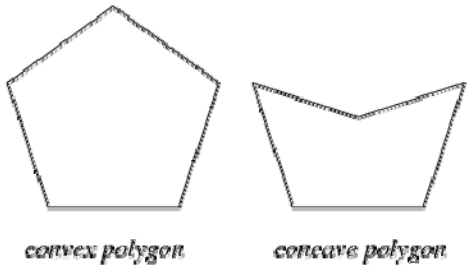
1.6 – Classifying Polygons

Polygon

A ***polygon*** is a closed plane figure with at least 3 sides. None of the sides may intersect at any other point than their endpoints.

Each endpoint of the sides of a polygon is called a ***vertex***. The plural of vertex is ***vertices***.

A polygon is ***convex*** if no line that contains a side of the polygons contains a point in the interior of the ploygon. A polygon that is not convex is called ***concave***.



Classifying Polygons by Number of Sides

# of Sides	Type of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n-gon

An ***equilateral*** polygon is one in which all sides are congruent.

An ***equiangular*** polygon is one in which all angles are congruent.

A ***regular*** polygon is a convex polygon that is both equilateral and equiangular.

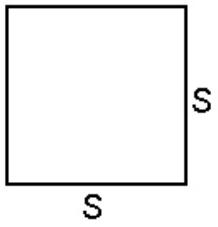
1.7 – Perimeter, Circumference and Area

Formulas

Square

$P = 4s$

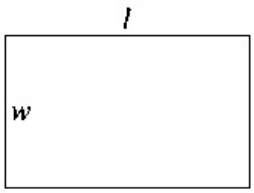
$A = s^2$



Rectangle

$P = 2l + 2w$

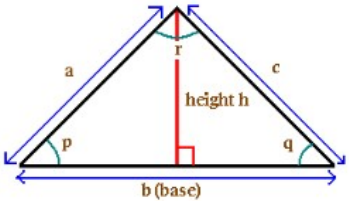
$A = l \bullet w$



Triangle

$P = a + b + c$

$A = \dfrac{b \bullet h}{2}$

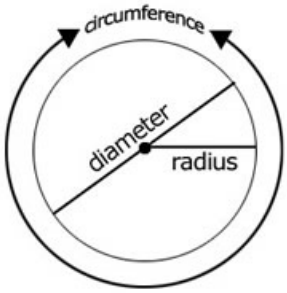


Circle

$C = 2 \bullet \pi \bullet r$

$A = \pi \bullet r^2$

$(\pi \approx 3.14)$



Square Unit Conversions

1 square foot = 144 square inches

1 square yard = 9 square feet

1 square meter = 10,000 square centimeters

1 square kilometer = 1,000,000 square meters