CP Statistics – Chapter 11 Notes: Testing a Claim

11.1: Significance Test Basics

Follow this plan when doing a significance test:

1. **State Hypotheses**: State the null and alternate hypotheses
2. **Check Conditions**: Check conditions for the appropriate test
3. **Perform Calculations**: Compute the test statistic and use it to find the p-value
4. **Interpret Results**: Use the p-value to state a conclusion, in context, in a sentence or two

Null and Alternate Hypotheses
The statement that is being tested is called the **null hypothesis (H_0)**. The significance test is designed to assess the strength of the evidence against the null hypothesis. Usually the null hypothesis is a statement of "no effect," "no difference," or no change from historical values.

The claim about the population that we are trying to find evidence for is called the **alternative hypothesis (H_a)**. Usually the alternate hypothesis is a statement of "an effect," "a difference," or a change from historical values.

Test Statistics
To assess how far the estimate is from the parameter, standardize the estimate. In many common situations, the test statistics has the form

$$\text{test statistic} = \frac{\textit{estimate} - \textit{parameter}}{\text{standard deviation of the estimate}}$$

P-value
The p-value of a test is the probability that we would get this sample result or one more extreme if the null hypothesis is true. The smaller the p-value is, the stronger the evidence against the null hypothesis provided by the data.

Statistical Significance
If the P-value (P) is as small as or smaller than alpha (α), we say that the data are statistically significant at level alpha. In general, use $\alpha = 0.05$ unless otherwise noted.

Interpretation – Decision

- If $P < \alpha$, REJECT H_o (strong evidence that H_0 is false)
- If $P > \alpha$, FAIL TO REJECT H_o (not enough evidence to state that H_0 is false)

Type I and Type II Errors
There are two types of errors that can be made using inferential techniques. In both cases, we get a sample that suggests we arrive at a given conclusion (either for or against H_0). Sometimes we get a bad sample that doesn't reveal the truth.

Here are the two types of errors:

- **Type I** – Rejecting the Ho when it is actually **true** (a false positive)
- **Type II** – Accepting the Ho when it is actually **false** (a false negative)

Be prepared to write, in sentence form, the meaning of a Type I and Type II error in the context of the given situation. Also, understand the consequences of each error, in context.

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11.2: Significance Test for a Population Proportion

Z-test for a Population Proportion (one-proportion z-test)

- **Hypotheses**: H_0: $p = p_0$; Ha: $p < p_0$ or $p > p_0$ or $p \neq p_0$

- **Conditions**:
  - SRS – does the data come from a random sample?
  - Normality – Are np_0 and $n(1-p_0)$ both at least 10?
  - Independence – is the population size at least 10 times greater than sample size?

- **Test-Statistic**: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ where $\hat{p}$ is the sample proportion

- **P-value**: The P-value is based on a normal z-distribution. This value can be estimated using Table A or found using an online normal probability calculator.

11.3: Significance Test for a Population Mean

T-test for a Population Mean

- **Hypotheses**: H_0: $\mu = \mu_0$; Ha: $\mu < \mu_0$ or $\mu > \mu_0$ or $\mu \neq \mu_0$

- **Conditions**:
  - SRS – does the data come from a random sample?
  - Normality – Is normality given or is there a large sample size ($n \geq 30$)?
  - Independence – is the population size at least 10 times greater than sample size?

- **Test-Statistic**: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ where *s* is the sample standard deviation

- **P-value**: The P-value is based on a t-distribution with $n-1$ degrees of freedom. This value can be estimated using Table C or found accurately using a *t*-test online calculator.