AP Statistics – Chapter 10 Notes: Estimating with Confidence

10.1 – Confidence Interval Basics

Confidence Interval A level *C* confidence interval is of the form *estimate* ± *margin of error*

The confidence level *C* gives us the probability that the interval will capture the true mean of the population. So when we use a 95% confidence level, we have a 95% chance of arriving at an interval containing the true population mean.

Conditions for Constructing a Confidence Interval for a Population Mean
The construction of a confidence interval for a population mean is appropriate when

- The data come from an SRS from the population of interest
- The sampling distribution of $\bar{x}$ is approximately normal*

* This can be achieved if n is large (at least 30) or if our statistic already adheres to a normal distribution (then, any sample size is OK)

Confidence Interval for a Population Mean
The form of the confidence interval for a population mean when the population standard deviation σ is known is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

Sample size for a desired margin of error
To determine the sample size (*n*) for a given margin of error m in a sample mean z interval use the formula

$$n = \left(\frac{z^* \bullet \sigma}{m} \right)^2$$

Remember, that we will always round up to ensure a smaller margin of error.

10.2 – Estimating a Population Mean

Conditions for Inference about a Population Mean

- **SRS** - Our data are a simple random sample (SRS) of size *n* from the population of interest. This condition is very important.
- **Normality** - Observations from the population have a normal distribution with mean μ and standard deviation σ. In practice, it is enough that the distribution be symmetric and single-peaked unless the sample is very small. Both μ and σ are unknown parameters.
- **Independence** - Population size is at least 10 times greater than sample size

Standard Error
When the standard deviation of a statistic is estimated from the data, the result is called the standard error of the statistic. The standard error of the sample mean is

$$\frac{s}{\sqrt{n}}$$

The form of the confidence interval for a population mean with $n-1$ degrees of freedom is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

10.2 – Estimating a Population Mean (continued)

Paired Differences t-interval

To compare the responses to the two treatments in a paired data design, apply the one-sample *t* procedures to the observed differences.

For example, suppose that pre and post test scores for 10 individuals in a summer reading program are:

Subject	1	2	3	4	5	6	7	8	9	10
Pre-test	25	31	28	27	30	31	22	18	24	30
Post-test	28	30	34	35	32	31	26	16	28	36
Difference	3	-1	6	8	2	0	4	-2	4	6

We would use the data in the differences row and perform one-sample *t* analysis on it.

Robustness of the t Procedures

Except in the case of small samples, the assumption that the data are an SRS from the population of interest is more important than the assumption that the population distribution is normal.

- *Sample size less than 15*. Use *t* procedures if the data are close to normal. If the data are clearly nonnormal or if outliers are present, do not use *t*.
- *Sample size at least 15*. The *t* procedures can be used except in the presence of outliers or strong skewness.
- *Large samples*. The *t* procedures can be used even for clearly skewed distributions when the sample is large, roughly $n \geq 30$.

10.3 – Estimating a Population Proportion

Conditions for Inference about a Population Proportion

- **SRS** - The data are a simple random sample (SRS) from the population of interest.
- **Normality** - Counts of successes and failures must be 10 or more.
- **Independence** - The population size is at least 10 times greater than the sample size

Standard Error of a Sample Proportion $\hat{p}$ is

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

One-Proportion z-interval

The form of the confidence interval for a population proportion is $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample size for a desired margin of error

To determine the sample size (n) for a given margin of error m in a 1-proportion z interval, use the formula

$$p^*(1-p^*)\left(\frac{z^*}{m}\right)^2$$

where p* = 0.5, unless another value is given.

Remember, that we will always round up to ensure a smaller margin of error.