

## **2.1: Measures of Relative Standing and Density Curves**

### **Density Curve**

A **density curve** is a curve that

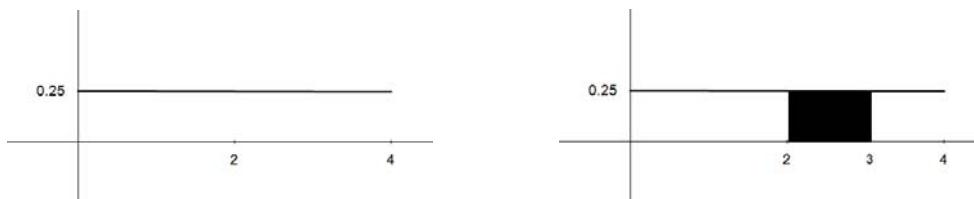
- is always on or above the horizontal axis, and
- has area exactly 1 underneath it.

A density curve describes the overall pattern of a distribution. The area under the curve and above any range of values is the proportion of all observations that fall in the range.

### **Example**

The density curve below left is a rectangle. The area underneath the curve is  $4 \cdot 0.25 = 1$ .

The figure on the right represents the proportion of data between 2 and 3 ( $1 \cdot 0.25 = 0.25$ ).



### **Median and Mean of a Density Curve**

- The **median** of a density curve is the **equal-areas point**, the point that divides the area under the curve in half.
- The **mean** of a density curve is the **balance point**, at which the curve would balance if made of solid material.
- The median and mean are the same for a symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.

### **Normal Distributions**

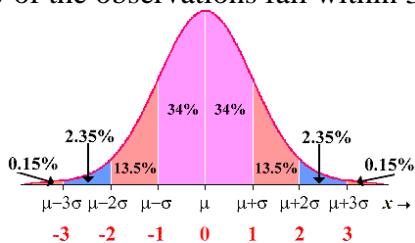
A normal distribution is a curve that is

- mound-shaped and symmetric
- based on a continuous variable
- adheres to the 68-95-99.7 Rule

### **The 68-95-99.7 Rule**

In the normal distribution with mean  $\mu$  and standard deviation  $\sigma$ :

- 68% of the observations fall within  $1\sigma$  of the mean  $\mu$ .
- 95% of the observations fall within  $2\sigma$  of the mean  $\mu$ .
- 99.7% of the observations fall within  $3\sigma$  of the mean  $\mu$ .



## 2.2: Normal Distributions

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### Standardizing and z-Scores

If  $x$  is an observation from a distribution that has mean  $\mu$  and standard deviation  $\sigma$ , the **standardized value of  $x$**  is

$$z = \frac{x - \mu}{\sigma}$$

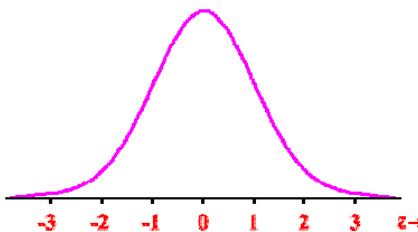
A standardized value is often called a **z-score**.

### Standard Normal Distribution

- The standard normal distribution is the normal distribution  $N(0, 1)$  with mean 0 and standard deviation 1.
- If a variable  $x$  has any normal distribution  $N(\mu, \sigma)$  with mean  $\mu$  and standard deviation  $\sigma$ , then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

has the standard normal distribution (see diagram below).



### The Standard Normal Table

**Table A** is a table of areas under the standard normal curve. The table entry for each value  $z$  is the area under the curve to the left of  $z$ .

### Standard Normal Calculations

Area to the left of $z$ ( $Z < z$ )	Area to the right of $z$ ( $Z > z$ )	Area between $z_1$ and $z_2$
 Area = Table Entry	 Area = 1 - Table Entry	 Area = difference between Table Entries for $z_1$ and $z_2$

### Inverse Normal Calculations

Working backwards from the area, we find  $z$ , then  $x$ . The value of  $z$  is found using Table A *in reverse*. The value of  $x$  is found, from  $z$ , using the formula below

$$x = \mu + z \cdot \sigma$$