CP Statistics Chapter 5 – Probability: What are the Chances?

Basic Probability Definitions and Rules

Probability

The **probability** of any outcome of a chance process is a number between 0 and 1 that describes the proportion of times the outcome would occur in a very long series of repetitions.

Sample Space

The **sample space *S*** of a chance process is the set of all possible outcomes.

Probability Models

Descriptions of chance behavior contain two parts:

A **probability model** is a description of some chance process that consists of two parts:

- a list of all possible outcomes
- a probability for each outcome.

For example: When a fair 6-sided die is rolled, the Sample Space is S = {1, 2, 3, 4, 5, 6}. The probability for a fair die would include the probabilities of these outcomes.

Outcome	**1**	**2**	**3**	**4**	**5**	**6**
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Event

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C*, and so on.

The Basic Rules of Probability

- For any event A, $0 \leq P(A) \leq 1$.
- If S is the sample space in a probability model, $P(S) = 1$.
- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$.
- **Multiplication rule for independent events:** If A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$.

General Addition Rule

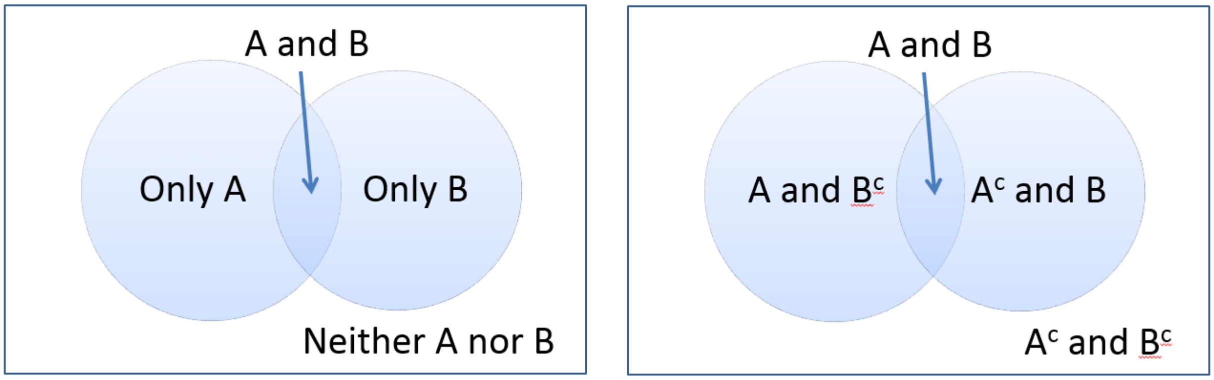
If A and B are any two events resulting from some chance process, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

General Multiplication Rule

If A and B are any two events resulting from some chance process, then
$P(A \text{ and } B) = P(A) \times P(B \mid A)$. Note that P (B | A) = the probability that B occurs, given that A has already occurred.

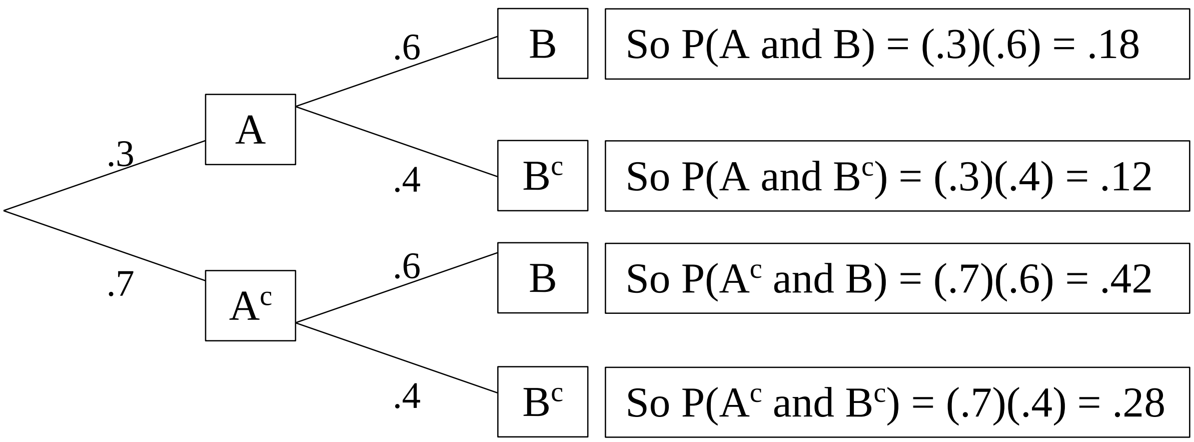
Venn Diagrams and Probability

Below shows how a Venn diagram explains the relationship between two events A and B.



Tree Diagrams and Probability

Example of a Tree Diagram with two events A and B, with P(A) = .3 and P(B) = .6



Conditional Probability

Conditional Probability
The probability that one event happens given that another event is already known to have happened is called a **conditional probability**.

Suppose we know that event A has happened. Then the probability that event *B* happens given that event *A* has happened is denoted by $P(B \mid A)$. The symbol “” is read as “given that,” so we read $P(B \mid A)$ as the probability that B occurs given that A has already occurred.

Calculating Conditional Probability

To find the conditional probability $P(A \mid B)$, use the formula

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Summary of Counting Methods

The Fundamental Counting Principle

If one event can occur in *a* ways, and another event can occur in *b* ways, then together these events can occur in $a \times b$ ways. This concept can be extended to three or more events (for example, $a \times b \times c$).

Example: If an outfit can be made using any one of 8 shirts, 4 pants and 5 hats, then there are $8 \times 4 \times 5 = 160$ different *shirt-pants-hat* outfits possible.

Factorial Rule for Simple Permutations

A collection of *n* different items can be arranged in *n*! different orders where $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Example: If a DJ wants to play 6 songs to end her set, how many different permutations (orders) are possible to choose from? There are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ different orders possible.

Rule for Permutations when all items are different

The number of permutations (or orders) of *r* items selected from a collection of *n* different available items (without replacement) is noted and calculated as

$$_nP_r = \frac{n!}{(n-r)!}$$

Example: If a DJ has 6 songs to choose from, but only enough time to play 4 of the songs to end her set, how many different permutations (orders) are possible to choose from? There are $6!/(6-2)! = 6!/4! = 30$ different orders possible to play 4 songs out of the 6 available.

Rule for Permutations when all items are NOT different

The number of permutations (or orders) of *r* items selected from a collection of *n* available items (without replacement) of which *a* items are alike, another *b* items are alike, another *c* items are alike, and so forth, is calculated as

$$\frac{n!}{a!\,b!\,c!}$$

Example: How many unique arrangements using the letters of the word ANTEATER are possible? There are 8 total letters in the word with 2 A's, 2 E's and 2 T's. So the number of unique rearrangements of these letters is $\frac{8!}{2!2!2!} = \frac{40320}{8} = 5040$ different arrangements.

Rule for Combinations when all items are different

The number of combinations (unordered collections) of *r* items selected from a collection of *n* different available items (without replacement) is noted and calculated as

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

Example: There are 20 members of a high school baseball team. Three players will be randomly selected for drug testing. How many different groups of 3 players are possible? Since the order of the players chosen does not matter, this is a combinations problem, not one of permutations. So the number of 3-player groups possible out of 20 players is $\frac{20!}{3!17!} = 1140$ different groups.