AP Statistics Practice Free Response Exam: 5 Questions, 65 minutes

STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 65 minutes on this part of the exam.
Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Caffeine, a chemical found in many popular beverages, is known for reducing fatigue. A student wanted to investigate the caffeine content in popular beverages, such as soft drinks, energy drinks, tea, and coffee. The following data collected by the student show the amounts of caffeine (in milligrams per 12-ounce serving) for twelve popular beverages.

   72 55 34 45 38 70 7.5 165 80 105 40 35

   (a) Construct an appropriate graphical display of the amounts of caffeine found in the twelve beverages.

   (b) Use the graph in part (a) to write a few sentences describing the distribution of caffeine content for the twelve beverages.

   (c) A 12-ounce cup of one popular gourmet coffee contains over 300 milligrams of caffeine. If this value was added to the data set of twelve numbers above, how would the mean and median of the data set above compare with the mean and median of the new data set with the thirteen numbers? Explain how this comparison could be made without performing any computations.

2. Members of the research and development division of a bicycle tire manufacturer are investigating tread life of rubber bicycle tires. They have suggested that a study be conducted to determine whether bicycle tires produced using a new synthetic rubber compound have a longer tread life than the tread life of bicycle tires produced using the standard rubber compound.

   A researcher in the division suggested the study be designed in the following way. Select 60 identical bicycles and randomly assign 30 of those bicycles to one group, A, and the rest to a second group, B. All 60 bicycles will be equipped with front tires produced using the standard rubber compound. However, the bicycles in group A will be equipped with rear tires produced using the new synthetic rubber compound, while the bicycles in group B will be equipped with rear tires produced using the standard rubber compound.

   A total of 60 bicyclists will be randomly selected from the population of students at a local university who regularly ride a bicycle. The 60 bicycles will be randomly assigned to the 60 students (with a different bicycle assigned to each student), and the students will be asked to ride the bicycles for a six-month period. At the end of the six-month period, the researcher will compare the mean amounts of rear tire tread wear for the bicycles in the two groups.

   (a) What type of design has been proposed for the study?

   What is the response variable in the design?

   (b) Other than using a larger sample size, describe a better design for the study than the one proposed by the researcher. Explain why your design is better.

   (c) For your design in part (b), identify a statistical test that could be conducted to determine whether tires produced using the new compound have longer tread life than tires produced using the standard compound. (You do not have to carry out the test.)

3. An important method for controlling the spread of the H6N2 influenza (bird flu) virus in chickens is having a procedure to determine whether chickens are infected with the virus. It is common to apply a procedure, called an ELISA test, to measure the concentration of anti–bird flu antibodies in a blood sample taken from a chicken. If the ELISA test reveals a high-enough concentration of antibodies, the chicken is said to test positive, and it is classified as infected with the virus. Otherwise, the chicken is said to test negative, and it is classified as not infected. However, the ELISA test is a complex procedure that is not always accurate. One type of mistake, a false positive result, occurs when the ELISA test gives a positive result for a chicken that is not infected with the virus. A second type of mistake, a false negative result, occurs when the ELISA test gives a negative result for an infected chicken.

   Considering the possibility of false positives and false negatives for tests on individual chickens, veterinarians have developed the following procedure for determining if the H6N2 virus is present in a large flock of chickens.

   - Randomly select 10 chickens from the flock.
   - Perform the ELISA test on a blood sample from each of the 10 chickens.
   - Conclude that the H6N2 virus is present in the flock if at least 3 out of the 10 chickens have positive ELISA test results.

   Suppose a veterinarian applies the procedure to a flock of 100,000 chickens at a commercial egg production farm. The ELISA test is known to have probability 0.05 of producing a false positive result and probability 0.10 of producing a false negative result for a single chicken.

   (a) If no chicken in the flock is infected with the H6N2 virus, what is the probability that the veterinarian will conclude that the H6N2 virus is not present in the flock? Show how you found your answer.

   (b) If no chicken in the flock is infected with the H6N2 virus, what is the probability that the veterinarian will conclude that the H6N2 virus is present in the flock? Show how you found your answer.

   (c) If every chicken in the flock is infected with the H6N2 virus, what is the probability that the veterinarian will conclude that the H6N2 virus is present in the flock? Show how you found your answer.

   (d) If 20 percent of the chickens in the flock are infected with the H6N2 virus and the other 80 percent are <u>not</u> infected, what is the probability that the veterinarian will conclude that the H6N2 virus is present in the flock? Show how you found your answer.

4. The department of parks and recreation of a certain city conducts summer programs for residents of its six districts. The summer programs include operating and maintaining community swimming pools in each of the districts as well as offering sports and recreational programs for school-age children, young adults, and older adults.

   The table below shows the proportion of households by district out of all households that participated in the summer programs, based on annual data that were collected from simple random samples each summer over a 10-year period, ending in the year 2000. The proportions are being used by the city for planning purposes and for more efficiently targeting the introduction of future programs.

District	A	B	C	D	E	F
Proportion of Households	0.32	0.12	0.10	0.27	0.05	0.14

   City leaders want to test if the proportions that are being used by the city are still valid. Data collected by a statistician from a simple random sample this past summer indicated that the following <u>number</u> of households participated in each district.

District	A	B	C	D	E	F
Number of Households	100	35	40	22	12	31

   (a) The statistician claims that the data for this past summer provide evidence that the proportions that are being used by the city are no longer valid. Give statistical evidence to justify the claim.

   (b) Which one of the six districts had the greatest change in participation since the year 2000 ? Use the information from part (a) to explain your choice.

5. Bone mineral density (BMD) is a measure of bone strength. It is defined as the ratio of bone mass to the cross-sectional area of the bone that is scanned, and it is expressed in units of grams per square centimeter (g/cm^2). Recent studies suggest that peak BMD in women is achieved between ages 15 and 40, and BMD declines after age 45. Decreased BMD is associated with increased risk of bone fracture.

In a recent study, the impact of regular physical exercise on women in differing stages of BMD development was examined. A simple random sample of 59 women between the ages of 41 and 45 and with no major health problems were enrolled in the study. The women were classified into one of the two following groups, based on their level of exercise activity.

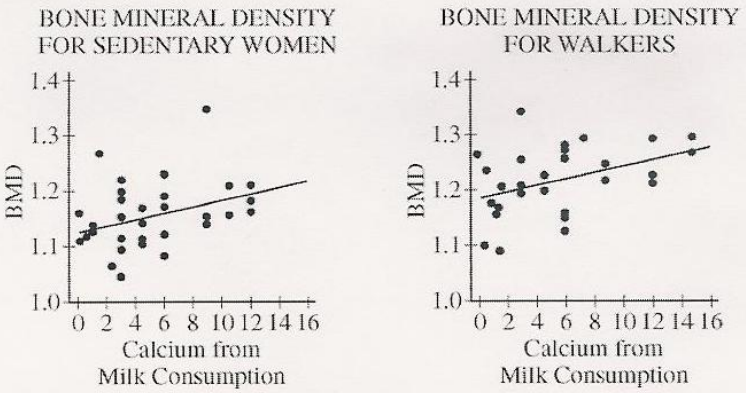
- Sedentary: minimal participation in physical exercise in the past three years (This group contained 31 women.)
- Walkers: walk at an aerobic pace at least 135 minutes per week during the past three years (This group contained 28 women.)

(a) The table below shows the mean BMD and corresponding standard deviation for each of the two groups of women.

Exercise Group	Number of Women	Mean BMD	Standard Deviation
Sedentary	31	1.151	0.068
Walkers	28	1.209	0.067

A *t*-test was conducted to compare the mean BMD levels for sedentary women and walkers. The results of the test showed a significant difference at the 0.01 level of significance. Based on the results, can it be concluded that converting sedentary women into walkers would necessarily increase their BMD levels? Explain.

(b) There was some concern that women in the two groups may have different dietary habits that could affect BMD. For example, higher intake of milk or other foods and supplements that provide additional calcium to the body could increase BMD. To examine this possibility, the researchers also asked each woman in the study to report on weekly milk consumption when she was age 20 through age 29. The data were then used to compute a value of calcium obtained from milk consumption for each woman in the study. BMD levels were plotted against the calcium intake from consumption of milk for women in each of the groups. The plots are shown below, with the least squares estimate of a regression line on each plot.



What do the plots indicate about the relationship between BMD and calcium intake from milk consumption from age 20 through age 29 ?

(c) The line graphs in part (b) suggest that sedentary women tend to have lower calcium intake from milk consumption than walkers do. Assuming that this is true, describe the impact, if any, that it would have on conclusions that can be reached from *t*-tests for comparing mean BMD levels for the two groups, such as those conducted in part (a).