

AP Statistics – Chapter 14 Practice Test: The Chi-Square Distributions

Part II, Free Response – Show all work and communicate completely and clearly.

- Computer software generated 500 random numbers that should look like they are from the uniform distribution on the interval 0 to 1. They are categorized into five groups: (1) less than or equal to 0.2 (2) greater than 0.2 and less than or equal to 0.4, (3) greater than 0.4 and less than or equal to 0.6, (4) greater than 0.6 and less than or equal to 0.8, and (5) greater than 0.8. The counts in the five groups are 113, 95, 108, 99, and 85, respectively.
 - The probabilities for these five intervals are all the same. What is this probability?
 - Compute the expected count for each interval for a sample of 500.
 - Perform the goodness of fit test and summarize your results. Show the entire calculation of the χ^2 statistic.
- The paper “Linkage Studies of the Tomato” (*Transactions of the Canadian Institute*, 1931) reported the following data on phenotypes resulting from crossing tall cut-leaf tomatoes with dwarf potato-leaf tomatoes. We wish to investigate if the frequencies below are consistent with genetic laws which state that the phenotypes should occur in the ratio 9:3:3:1.

	Phenotype			
	Tall Cut- leaf	Tall Pot.- leaf	Dwarf Cut- leaf	Dwarf Pot.- leaf
Frequency	926	288	293	104

- State appropriate null and alternative hypotheses for investigating the genetic model.
 - Verify conditions for performing inference in this setting.
 - Compute the expected counts for this data.
 - Calculate the test statistic and the P -value.
 - What conclusion would you draw?
- A study of the career plans of young women and men sent questionnaires to all 722 members of the senior class in the College of Business Administration at the University of Illinois. One question asked which major within the business program the student had chosen. Here are the data from the students who responded:

	Female	Male
Accounting	68	56
Administration	91	40
Economics	5	6
Finance	61	59

This is an example of a single sample classified according to two categorical variables (gender and major).

- Verify that the expected cell counts satisfy the requirement for use of chi-square.
- Compute the expected count for the female economics cell of the table. **SHOW WORK.**
- Test the null hypothesis that there is no relationship between the gender of students and their choice of major. Give the χ^2 statistic and the P -value.
- State your conclusion in a sentence or two.

AP Statistics – Chapter 14 Practice FR Solutions

1. Goodness of Fit Test – Equal Proportions

- a) Since there are 5 groups, the probability is $1/5 = 0.2$
- b) Since there are 500 numbers generated, expected counts are $500(0.2) = 100$
- c) The formula for χ^2 statistic is $\chi^2 = \sum \frac{(\text{obs} - \text{exp})^2}{\text{exp}}$. For this data, the calculation is
- $$\frac{(113-100)^2}{100} + \frac{(95-100)^2}{100} + \frac{(108-100)^2}{100} + \frac{(99-100)^2}{100} + \frac{(85-100)^2}{100} = 4.84$$

There are 4 degrees of freedom here, so the p-value is .3041.

Conclusion: Since the p-value is high ($p > .05$), we fail to reject the null hypothesis. So we can conclude that the random numbers are uniformly distributed among the 5 groups.

2. Goodness of Fit Test – Unequal Proportions

- a) Ho: Data matches the given ratios Ha: Data does not match the given ratios
- b) All expected counts are greater than 5. We also require that data is randomly selected.
- c) For the ratios 9:3:3:1, we have 16 total parts ($9+3+3+1=16$). So we divide the total 1611 by 16 which equals 100.69. So using the ratios given, a 9:3:3:1 the expected counts would be $9(100.69)$, $3(100.69)$, $3(100.69)$ and $1(100.69) = 906.19$, 302.06 , 302.06 and 100.69 .
- d) There are 3 degrees of freedom here, so $\chi^2 = 1.468$ and the p-value is .6900.
- e) Since the p-value is high ($p > .05$), we fail to reject the null hypothesis. So we can conclude that the numbers in our sample are consistent with genetic law (matches the expected ratios).

3. Test of Association Between Categorical Variables – Two-Way Table

- a) Only one expected count is less than 5. The count for (*Men, Economics*) = 4.59. Since this is only 1 out of 8 counts (12.5%) we are OK to use the chi-square test.

b) Expected counts are found by $\frac{(\text{row tot})(\text{col tot})}{\text{table tot}} = \frac{(11)(225)}{386} = 6.412$.

- c) There are 3 degrees of freedom here, so $\chi^2 = 10.827$ and the p-value is .0127
- d) Since the p-value is less than .05, we reject the null hypothesis. So we can conclude that there is a relationship between gender and their choice of major. Stated another way, men and women do not choose majors in the same way.