

## AP Statistics Chapter 7/8 – Discrete, Binomial and Geometric Rand. Vars.

### 7.1: Discrete Random Variables

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#### Random Variable

A random variable is a variable whose value is a numerical outcome of a random phenomenon.

#### Discrete Random Variable

A discrete random variable  $X$  has a *countable* number of possible values. Generally, these values are limited to integers (whole numbers). The probability distribution of  $X$  lists the values and their probabilities.

<b>Value of X</b>	<b><math>x_1</math></b>	<b><math>x_2</math></b>	<b><math>x_3</math></b>	<b>...</b>	<b><math>x_k</math></b>
<b>Probability</b>	<b><math>p_1</math></b>	<b><math>p_2</math></b>	<b><math>p_3</math></b>	<b>...</b>	<b><math>p_k</math></b>

The probabilities  $p_i$  must satisfy two requirements:

1. Every probability  $p_i$  is a number between 0 and 1.
2.  $p_1 + p_2 + \dots + p_k = 1$

Find the probability of any event by adding the probabilities  $p_i$  of the particular values  $x_i$  that make up the event.

#### Continuous Random Variable

A continuous random variable  $X$  takes all values in an interval of numbers and is *measurable*.

### 7.2: The Mean of a Discrete Random Variable

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#### Mean Of A Discrete Random Variable

Suppose that  $X$  is a discrete random variable whose distribution is

<b>Value of X</b>	<b><math>x_1</math></b>	<b><math>x_2</math></b>	<b><math>x_3</math></b>	<b>...</b>	<b><math>x_k</math></b>
<b>Probability</b>	<b><math>p_1</math></b>	<b><math>p_2</math></b>	<b><math>p_3</math></b>	<b>...</b>	<b><math>p_k</math></b>

To find the **mean** of  $X$ , multiply each possible value by its probability, then add all the products:

$$\mu_X = \sum_{i=1}^k x_i \cdot p_i = x_1 \cdot p_1 + x_2 \cdot p_2 + \dots + x_k \cdot p_k$$

#### Law Of Large Numbers

Draw independent observations at random from any population with finite mean  $\mu$ . As the number of observations drawn increases, the mean of the observed values eventually approaches the mean  $\mu$ .

## 8.1: The Binomial Distributions

A **binomial probability distribution** occurs when the following requirements are met.

1. Each observation falls into one of just two categories – call them “success” or “failure.”
2. The procedure has a fixed number of trials – we call this value  $n$ .
3. The observations must be *independent* – result of one does not affect another.
4. The probability of success – call it  $p$  - remains the same for each observation.

### Notation for binomial probability distribution

$n$  denotes the number of fixed trials

$k$  denotes the number of successes in the  $n$  trials

$p$  denotes the probability of success

$1 - p$  denotes the probability of failure

#### Binomial Probability Formula

$$P(X = k) = \frac{n!}{k!(n-k)!} (p)^k (1-p)^{n-k}$$

### How to use the TI-83/4 to compute binomial probabilities \*

There are two binomial probability functions on the TI-83/84, *binompdf* and *binomcdf*

*binompdf* is a *probability distribution function* and determines  $P(X = k)$

*binomcdf* is a *cumulative distribution function* and determines  $P(X \leq k)$

\*Both functions are found in the DISTR menu (2<sup>nd</sup>-VARS)

Probability	Calculator Command	Example (assume $n = 4, p = .8$ )
$P(X = k)$	$binompdf(n, p, k)$	$P(X = 3) = binompdf(4, .8, 3)$
$P(X \leq k)$	$binomcdf(n, p, k)$	$P(X \leq 3) = binomcdf(4, .8, 3)$
$P(X < k)$	$binomcdf(n, p, k - 1)$	$P(X < 3) = binomcdf(4, .8, 2)$
$P(X > k)$	$1 - binomcdf(n, p, k)$	$P(X > 3) = 1 - binomcdf(4, .8, 3)$
$P(X \geq k)$	$1 - binomcdf(n, p, k - 1)$	$P(X \geq 3) = 1 - binomcdf(4, .8, 2)$

### Mean (expected value) of a Binomial Random Variable

Formula:  $\mu = np$       Meaning: Expected number of successes in  $n$  trials (think *average*)

Example: *Suppose you are a 80% free throw shooter. You are going to shoot 4 free throws.*

For  $n = 4, p = .8, \mu = (4)(.8) = 3.2$ , which means we expect 3.2 makes out of 4 shots, on average

## 8.2: The Geometric Distributions

A **geometric probability distribution** occurs when the following requirements are met.

1. Each observation falls into one of just two categories – call them “success” or “failure.”
2. The observations must be *independent* – result of one does not affect another.
3. The probability of success – call it  $p$  - remains the same for each observation.
4. The variable of interest is the number of trials required to obtain the first success.\*

\* As such, the geometric is also called a “waiting-time” distribution

### Notation for geometric probability distribution

$n$  denotes the number of trials required to obtain the first success

$p$  denotes the probability of success

$1 - p$  denotes the probability of failure

### Geometric Probability Formula

$$P(X = n) = (1 - p)^{n-1}(p)$$

### How to use the TI-83/4 to compute geometric probabilities \*

There are two geometric probability functions on the TI-83/84, *geompdf* and *geomcdf*

*geompdf* is a *probability distribution function* and determines  $P(X = n)$

*geomcdf* is a *cumulative distribution function* and determines  $P(X \leq n)$

\*Both functions are found in the DISTR menu (2<sup>nd</sup>-VARS)

Probability	Calculator Command	Example (assume $p = .8$ , $n = 3$ )
$P(X = n)$	<i>geompdf</i> ( $p$ , $n$ )	$P(X = 3) = \text{geompdf}(.8, 3)$
$P(X \leq n)$	<i>geomcdf</i> ( $p$ , $n$ )	$P(X \leq 3) = \text{geomcdf}(.8, 3)$
$P(X < n)$	<i>geomcdf</i> ( $p$ , $n-1$ )	$P(X < 3) = \text{geomcdf}(.8, 2)$
$P(X > n)$	$1 - \text{geomcdf}(p, n)$	$P(X > 3) = 1 - \text{geomcdf}(.8, 3)$
$P(X \geq n)$	$1 - \text{geomcdf}(p, n-1)$	$P(X \geq 3) = 1 - \text{geomcdf}(.8, 2)$

### Mean (expected value) of a Geometric Random Variable

Formula:  $\mu = \frac{1}{p}$       Meaning: Expected number of  $n$  trials to achieve first success (*average*)

Example: *Suppose you are a 80% free throw shooter. You are going to shoot until you make.*

For  $p = .8$ ,  $\mu = \frac{1}{.8} = 1.25$ , which means we expect to take 1.25 shots, on average, to make first