AP Statistics Chapter 7/8 – Discrete, Binomial and Geometric Rand. Vars.

7.1: Discrete Random Variables

Random Variable
A random variable is a variable whose value is a numerical outcome of a random phenomenon.

Discrete Random Variable
A discrete random variable *X* has a *countable* number of possible values. Generally, these values are limited to integers (whole numbers). The probability distribution of X lists the values and their probabilities.

Value of X	x_1	x_2	x_3	…	x_k
Probability	p_1	p_2	p_3	…	p_k

The probabilities p_i must satisfy two requirements:

1. Every probability p_i is a number between 0 and 1.
2. $p_1 + p_2 + \ldots + p_k = 1$

Find the probability of any event by adding the probabilities p_i of the particular values x_i that make up the event.

Continuous Random Variable
A continuous random variable *X* takes all values in an interval of numbers and is *measurable*.

7.2: The Mean of a Discrete Random Variable

Mean Of A Discrete Random Variable
Suppose that X is a discrete random variable whose distribution is

Value of X	x_1	x_2	x_3	…	x_k
Probability	p_1	p_2	p_3	…	p_k

To find the **mean** of X, multiply each possible value by its probability, then add all the products:

$$\mu_X = \sum_{i=1}^{k} x_i \bullet p_i = x_1 \bullet p_1 + x_2 \bullet p_2 + \cdots + x_k \bullet p_k$$

Law Of Large Numbers
Draw independent observations at random from any population with finite mean μ. As the number of observations drawn increases, the mean of the observed values eventually approaches the mean μ.

8.1: The Binomial Distributions

A **binomial probability distribution** occurs when the following requirements are met.

1. Each observation falls into one of just two categories – call them "success" or "failure."
2. The procedure has a fixed number of trials – we call this value *n*.
3. The observations must be *independent* – result of one does not affect another.
4. The probability of success – call it *p* - remains the same for each observation.

Notation for binomial probability distribution

n denotes the number of fixed trials

k denotes the number of successes in the *n* trials

p denotes the probability of success

1 – p denotes the probability of failure

Binomial Probability Formula

$$P(X=k)=\frac{n!}{k!(n-k)!}(p)^k(1-p)^{n-k}$$

How to use the TI-83/4 to compute binomial probabilities *

There are two binomial probability functions on the TI-83/84, *binompdf* and *binomcdf*

binompdf is a *probability distribution function* and determines $P(X=k)$

binomcdf is a *cumulative distribution function* and determines $P(X\le k)$

*Both functions are found in the DISTR menu (2^{nd}-VARS)

Probability	Calculator Command	Example (assume n = 4, p = .8)
$P(X=k)$	*binompdf(n, p, k)*	$P(X=3)$ = ***binompdf(4, .8, 3)***
$P(X\le k)$	*binomcdf(n, p, k)*	$P(X\le 3)$ = ***binomcdf(4, .8, 3)***
$P(X<k)$	*binomcdf(n, p, k - 1)*	$P(X<3)$ = ***binomcdf(4, .8, 2)***
$P(X>k)$	*1 – binomcdf(n, p, k)*	$P(X>3)$ = **1 – *binomcdf(4, .8, 3)***
$P(X\ge k)$	*1 – binomcdf(n, p, k - 1)*	$P(X\ge 3)$ = **1 – *binomcdf(4, .8, 2)***

Mean (expected value) of a Binomial Random Variable

Formula: $\mu=np$ Meaning: Expected number of successes in *n* trials (think *average*)

Example: *Suppose you are a 80% free throw shooter. You are going to shoot 4 free throws.*

For *n* = 4, *p* = .8, $\mu=(4)(.8)=3.2$, which means we expect 3.2 makes out of 4 shots, on average

8.2: The Geometric Distributions

A **geometric probability distribution** occurs when the following requirements are met.

1. Each observation falls into one of just two categories – call them "success" or "failure."
2. The observations must be *independent* – result of one does not affect another.
3. The probability of success – call it *p* - remains the same for each observation.
4. The variable of interest is the number of trials required to obtain the first success.*

* As such, the geometric is also called a "waiting-time" distribution

Notation for geometric probability distribution

n denotes the number of trials required to obtain the first success

p denotes the probability of success

1 – p denotes the probability of failure

Geometric Probability Formula
$P(X = n) = (1-p)^{n-1}(p)$

How to use the TI-83/4 to compute geometric probabilities *

There are two geometric probability functions on the TI-83/84, *geometpdf* and *geometcdf*

geometpdf is a *probability distribution function* and determines $P(X = n)$

geometcdf is a *cumulative distribution function* and determines $P(X \le n)$

*Both functions are found in the DISTR menu (2^{nd}-VARS)

Probability	Calculator Command	Example (assume p = .8, n = 3)
$P(X = n)$	*geometpdf (p, n)*	$P(X = 3)$ = ***geometpdf(.8, 3)***
$P(X \le n)$	*geometcdf(p, n)*	$P(X \le 3)$ = ***geometcdf(.8, 3)***
$P(X < n)$	*geometcdf(p, n-1)*	$P(X < 3)$ = ***geometcdf(.8, 2)***
$P(X > n)$	*1 – geometcdf(p, n)*	$P(X > 3)$ = **1 – *geometcdf(.8, 3)***
$P(X \ge n)$	*1 – geometcdf(p, n-1)*	$P(X \ge 3)$ = **1 – *geometcdf(.8, 2)***

Mean (expected value) of a Geometric Random Variable

Formula: $\mu = \frac{1}{p}$ Meaning: Expected number of *n* trials to achieve first success (*average*)

Example: *Suppose you are a 80% free throw shooter. You are going to shoot until you make.*

For *p* = .8, $\mu = \frac{1}{.8} = 1.25$, which means we expect to take 1.25 shots, on average, to make first