Probability Terms and Rules

You need to know all these terms and rules and be able to apply then. Your book has a good source of exercises and this packet supplements that.

Probability – the study of random phenomena. The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

The probability of an event is a number from 0 to 1 inclusive.

Probability 0	Probability 0.5	Probability 1
It cannot happen	As likely to happen as not	It must happen

Sample Space - the set of all possible outcomes of an experiment with random outcome

Example: Experiment: rolling a die: Sample space = $\{1,2,3,4,5,6\}$

Since the sample space is everything that can occur, adding the probabilities of everything in the sample Space must add up to 1.

Event: an outcome or outcomes in a sample space:

Example: Experiment: rolling a die. Event: result is even.

Probability of an event: $\dfrac{\text{number of ways the event can occur}}{\text{number of members in the sample space}}$

Example: Event: rolling a die. Probability (rolling an even number) = $\dfrac{3}{6}=\dfrac{1}{2}$

Multiplication principal: If you can do a task in *m* ways and a second task in *n* ways, then the number of ways that both tasks can be done is $a\times b$ ways

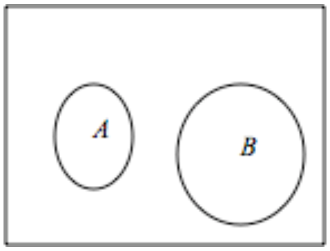
Example: Events: Rolling a die then tossing a coin. The number of elements in the sample space is 6(2) = 12.

Complement of an event: The complement of even A is the event that A does not occur:

Formula: $P\left(A^C\right)=1-P(A)$

Example: Experiment: Rolling a die: A: rolling a number < 3. $P(A)=\dfrac{2}{6}$ so $P\left(A^C\right)=1-P(A)=\dfrac{4}{6}=\dfrac{2}{3}$

Disjoint events: Two events A and B are disjoint (sometimes called **mutually exclusive)** if they have no outcomes in common and can never happen simultaneously. This can be shown in the **Venn Diagram** below. Notice that A and B have nothing in common.



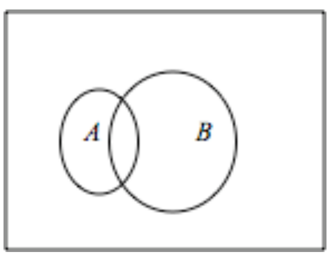
Formula: $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

Example: A cooler contains 20 bottles made up of 8 Cokes, 5 Pepsis and 7 waters. The Probability of choosing a Coke or Pepsi is $\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$.

General Addition rule. If two (or more) events are not disjoint the above formula doesn't work. The rule is:

Formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

The Venn Diagram below shows this relationship: If we simply add the probabilities, we will be adding the middle section twice. So we have to subtract one of them.



Example: In a class, there are 12 boys made up of 8 Seniors and 4 Juniors . There are also 8 girls, made up of 3 Seniors and 5 Juniors. Find the probability of choosing a boy or a Senior.

Note that choosing a boy and choosing a Senior are not disjoint (they can occur simultaneously). So the Probability of choosing a boy or a senior = P(Boy) + P(Senior) – P(Senior boy) = $\frac{12}{20} + \frac{11}{20} - \frac{8}{20} = \frac{15}{20} = \frac{3}{4}$.

A chart best shows this relationship:

	Boy	Girl	Total
Senior	8	3	11
Junior	4	5	9
Total	12	8	

Independence: Two events are independent if knowing one occurs doesn't change the probability of the other occurring. This is the non-mathematical definition.

Example: Tossing one coin and then another coin are independent events. The result of the second coin toss has nothing to do with the results of the first coin toss.

Proving independence is difficult. For instance is making a second foul shot in basketball independent of the first foul shot? Arguments?

Mathematical way of proving independence: If two events are independent then $P(A)\cdot P(B) = P(A \text{ and } B)$. Also, if $P(A)\cdot P(B) = P(A \text{ and } B)$, then the two events are independent: This is called the mathematical rule. If events are independent, to find the probability of them all happening, you may multiply the probabilities.

Example: In the problem above, are choosing a Senior and choosing a boy independent?

If so, $P(\text{Senior})\cdot P(\text{Boy}) = P(\text{Senior boy})$: $\frac{11}{20}\cdot\frac{12}{20} = \frac{8}{20}$? $.33 \neq .40$ so events are not independent

Choosing a boy influences the probability of choosing a Senior.

Conditional Probability: The probability of one event happening, given that another event has happened.

The way this is written is $P(B \mid A)$ which means the probability of *B* occurring given that *A* occurred.

Again, let's use this example:

	Boy	Girl	Total
Senior	8	3	11
Junior	4	5	9
Total	12	8	20

To find the $P(\text{Boy} \mid \text{Senior})$ (the probability of choosing a boy given that we chose a senior), we look at the chart and see that there are 11 seniors and 8 of them are boys. So $P(\text{Boy} \mid \text{Senior}) = \frac{8}{11}$

To find the $P(\text{Senior} \mid \text{Boy})$ (the probability of choosing a Senior given that we chose a boy), We look at the chart and see that there are 12 boys and 8 of them are Seniors. So $P(\text{Senior} \mid \text{Boy}) = \frac{8}{12} = \frac{2}{3}$

The formula for $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$. Using this formula:

$$P(\text{Boy} \mid \text{Senior}) = \frac{P(\text{Senior Boy})}{P(\text{Senior})} = \frac{\frac{8}{20}}{\frac{11}{20}} = \frac{8}{11}$$

$$P(\text{Senior} \mid \text{Boy}) = \frac{P(\text{Senior Boy})}{P(\text{Boy})} = \frac{\frac{8}{20}}{\frac{12}{20}} = \frac{8}{12} = \frac{2}{3}$$