AP Statistics – Chapter 9 Notes: Testing a Claim

9.1: Significance Test Basics

Null and Alternate Hypotheses

The statement that is being tested is called the **null hypothesis (H_0)**. The significance test is designed to assess the strength of the evidence against the null hypothesis. Usually the null hypothesis is a statement of "no effect," "no difference," or no change from historical values.

The claim about the population that we are trying to find evidence for is called the **alternative hypothesis (H_a)**. Usually the alternate hypothesis is a statement of "an effect," "a difference," or a change from historical values.

Test Statistics

To assess how far the estimate is from the parameter, standardize the estimate. In many common situations, the test statistics has the form

$$\text{test statistic} = \frac{\textit{estimate} - \textit{parameter}}{\text{standard deviation of the estimate}}$$

P-value

The p-value of a test is the probability that we would get this sample result or one more extreme if the null hypothesis is true. The smaller the p-value is, the stronger the evidence against the null hypothesis provided by the data.

Statistical Significance

If the P-value is as small as or smaller than alpha, we say that the data are statistically significant at level alpha. In general, use alpha = 0.05 unless otherwise noted.

A Plan for Carrying out a Significance Test:

1. *Hypotheses*: State the null and alternate hypotheses
2. *Conditions*: Check conditions for the appropriate test
3. *Calculations*: Compute the test statistic and use it to find the p-value
4. *Interpretation*: Use the p-value to state a conclusion, in context, in a sentence or two

Type I and Type II Errors

There are two types of errors that can be made using inferential techniques. In both cases, we get a sample that suggests we arrive at a given conclusion (either for or against H_0). Sometimes we get a bad sample that doesn't reveal the truth.

Here are the two types of errors:

Type I – Rejecting the Ho when it is actually **true** (a false positive)
Type II – Accepting the Ho when it is actually **false** (a false negative)

Be prepared to write, in sentence form, the meaning of a Type I and Type II error in the context of the given situation. The **probability of a Type I error** is the same as alpha, the significance level. You will not be asked to find the probability of a Type II error.

9.2: Tests about a Population Proportion

Z-test for a Population Proportion (one-proportion z-test)

1. **Hypotheses**: **H_0**: $p = p_0$; **Ha**: $p < p_0$ or $p > p_0$ or $p \neq p_0$
2. **Conditions**:
   - **Random** – does the data come from a random sample?
   - **Independent** – is the sample size less than 10% of the population size?
   - **Normal** – Are np_0 and $n(1-p_0)$ both at least 10?
3. **Test-Statistic**: $z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ where $\hat{p}$ is the sample proportion

   P-value: The P-value is based on a normal z-distribution. This value can be estimated using Table A or found accurately using the *1-Prop Z-test* function on your calculator

4. **Conclusion:** If P < α, then Reject the H_0, otherwise Fail to Reject H_0.

9.3: Tests about a Population Mean

T-test for a Population Mean

1. **Hypotheses**: **H_0**: $\mu = \mu_0$; **Ha**: $\mu < \mu_0$ or $\mu > \mu_0$ or $\mu \neq \mu_0$
2. **Conditions**:
   - **Random** – does the data come from a random sample?
   - **Independent** – is the sample size less than 10% of the population size?
   - **Normal** – Is it given or is there a large sample size ($n \geq 30$)?
3. **Test-Statistic**: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ where s is the sample standard deviation

   P-value: The P-value is based on a t-distribution with $n-1$ degrees of freedom. This value can be estimated using Table C or found accurately using the *T-test* function on your calculator

4. **Conclusion:** If P < α, then Reject the H_0, otherwise Fail to Reject H_0.

Paired Differences T-test

To compare the responses to the two treatments in a paired data design, apply the one-sample *t* procedures to the observed differences.

For example, suppose that pre and post test scores for 10 individuals in a summer reading program are:

Subject	1	2	3	4	5	6	7	8	9	10
Pre-test	25	31	28	27	30	31	22	18	24	30
Post-test	28	30	34	35	32	31	26	16	28	36
Difference	3	-1	6	8	2	0	4	-2	4	6

We would use the data in the differences row and perform one-sample *t* analysis on it.