

## AP Statistics – Chapter 8 Notes: Estimating with Confidence

### 8.1 – Confidence Interval Basics

#### Point Estimate

A *point estimator* is a statistic that provides an estimate of a population parameter. The value of that statistic from a sample is called a *point estimate*.

#### The Idea of a Confidence Interval

A *C% confidence interval* gives an interval of plausible values for a parameter. The interval is calculated from the data and has the form: **point estimate  $\pm$  margin of error**

The difference between the point estimate and the true parameter value will be less than the margin of error in C% of all samples.

The *confidence level C* gives the overall success rate of the method for calculating the confidence interval. That is, in C% of all possible samples, the method would yield an interval that captures the true parameter value.

#### Interpreting Confidence Intervals

To interpret a C% confidence interval for an unknown parameter, say, “We are C% confident that the interval from \_\_\_\_\_ to \_\_\_\_\_ captures the actual value of the [population parameter in context].”

#### Interpreting Confidence Levels

To say that we are *95% confident* is shorthand for “If we take many samples of the same size from this population, about 95% of them will result in an interval that captures the actual parameter value.”

### 8.2 – Estimating a Population Proportion

#### Conditions for Inference about a Population Proportion

- **Random Sample** - The data are a random sample from the population of interest.
- **10% Rule** - The sample size is no more than 10% of the population size:  $n \leq \frac{1}{10} N$
- **Large Counts** - Counts of successes and failures must be 10 or more:  $n\hat{p} \geq 10$  and  $n(1 - \hat{p}) \geq 10$

#### Standard Error of a Sample Proportion $\hat{p}$ is

$$\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

#### One-Proportion z-interval

The form of the confidence interval for a population proportion is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

#### Sample size for a desired margin of error

To determine the sample size ( $n$ ) for a given margin of error  $m$  in a 1-proportion z interval, use formula

$$n = p^* (1 - p^*) \left( \frac{z^*}{m} \right)^2$$

where  $p^* = 0.5$ , unless another value is given. Remember, that we will always round up to ensure a slightly smaller margin of error than is required.

## 8.3 – Estimating a Population Mean

### Conditions for Inference about a Population Mean

- **Random Sample** - The data are a random sample from the population of interest.
- **10% Rule** - The sample size is no more than 10% of the population size:  $n \leq \frac{1}{10} N$
- **Large Counts/Normality** – If the sample size is large ( $n \geq 30$ ), then we can assume normality for any shape of distribution. When sample is smaller than 30, the  $t$  procedures can be used except in the presence of outliers or strong skewness. Construct a quick graph of the data to make an assessment.

### Standard Error

When the standard deviation of a statistic is estimated from the data, the result is called the *standard error* of the statistic. The standard error of the sample mean is

$$\frac{s}{\sqrt{n}}$$

### One-Sample t-Interval for Estimating a Population Mean

The form of the confidence interval for a population mean with  $n - 1$  degrees of freedom is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

### Paired Differences t-interval

To compare the responses to the two treatments in a paired data design, apply the one-sample  $t$  procedures to the observed differences.

For example, suppose that pre and post test scores for 10 individuals in a summer reading program are:

<b>Subject</b>	1	2	3	4	5	6	7	8	9	10
<b>Pre-test</b>	25	31	28	27	30	31	22	18	24	30
<b>Post-test</b>	28	30	34	35	32	31	26	16	28	36
<b>Difference</b>	3	-1	6	8	2	0	4	-2	4	6

We would then use the data in the “difference” row and perform one-sample  $t$  analysis on it.