AP Statistics Chapter 5 – Probability: What are the Chances?

5.1: Randomness, Probability and Simulation

Probability

The **probability** of any outcome of a chance process is a number between 0 and 1 that describes the proportion of times the outcome would occur in a very long series of repetitions.

Simulation

The imitation of chance behavior, based on a model that accurately reflects the situation, is called a **simulation**.

Performing of a Simulation – The 4-Step Process

1. **State**: Ask a question of interest about some chance process.
2. **Plan**: Describe how to use a chance device to imitate one repetition of the process. Tell what you will record at the end of each repetition.
3. **Do**: Perform many repetitions of the simulation.
4. **Conclude**: Use the results of your simulation to answer the question of interest.

5.2: Probability Rules

Sample Space

The **sample space *S*** of a chance process is the set of all possible outcomes.

Probability Models

Descriptions of chance behavior contain two parts:

A **probability model** is a description of some chance process that consists of two parts:

- a sample space S and
- a probability for each outcome.

For example: When a fair 6-sided die is rolled, the Sample Space is S = {1, 2, 3, ,4,5, 6}. The probability for a fair die would include the probabilities of these outcomes, which are all the same.

Outcome	**1**	**2**	**3**	**4**	**5**	**6**
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Event

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

For example: For the probability model above we might define event A = die roll is odd. The elements of the sample space S that fits this event are {1, 3, 5}. The probability of the event A, written as P(A) is the 3/6 or ½. So we would write P(A) = 0.5, in decimal form.

The Basic Rules of Probability

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model, $P(S) = 1$.
- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$. Also be familiar with the notation: $\boldsymbol{P(A \cup B)}$.

Mutually Exclusive Events

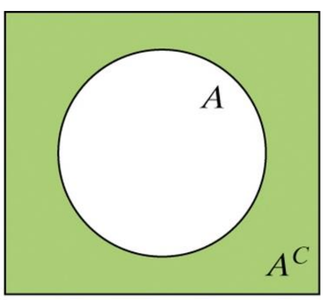
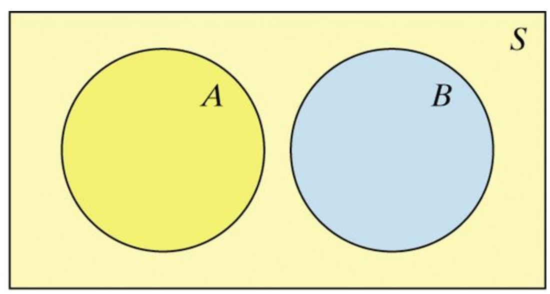
Two events A and B are **mutually exclusive** (or **disjoint**) if they have no outcomes in common and so can never occur together—that is, if $P(A \text{ and } B) = 0$. Alternate notation: $\boldsymbol{P(A \cap B)}$.

For example: Using a deck of playing cards and drawing a card at random, the events A = card is a King, and B = card is a Queen are mutually exclusive because a single card cannot be both a King and a Queen. Thus we can calculate the probability of A or B as the sum of their individual probabilities - P(A or B) = P(A) + P(B).

General Addition Rule

If A and B are any two events resulting from some chance process, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Venn Diagrams and Probability

<table>
<tr>
<td>The complement A^c contains exactly the outcomes that are not in A.

</td>
<td>The events A and B are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

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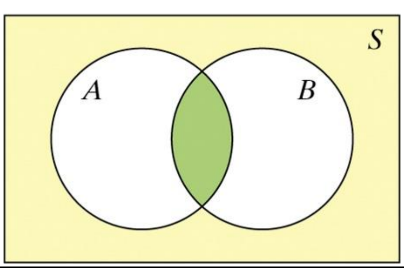
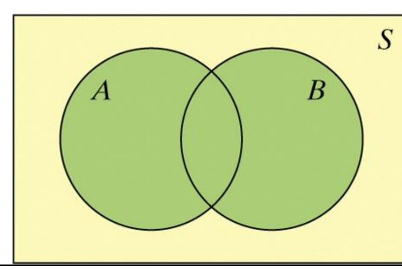
<tr>
<td>The intersection of events A and B ($A \cap B$) is the set of all outcomes in both events A and B.
$\boldsymbol{A \cap B}$

</td>
<td>The union of events A and B ($A \cup B$) is the set of all outcomes in either event A or B.
$\boldsymbol{A \cup B}$

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5.3: Conditional Probability and Independence

Conditional Probability
The probability that one event happens given that another event is already known to have happened is called a **conditional probability**.

Suppose we know that event A has happened. Then the probability that event *B* happens given that event *A* has happened is denoted by $P(B \mid A)$. The symbol "" is read as "given that," so we read $P(B \mid A)$ as the probability that B occurs given that A has already occurred.

Calculating Conditional Probability

To find the conditional probability $P(A \mid B)$, use the formula

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The conditional probability $P(B \mid A)$ is given by

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

The General Multiplication Rule
The probability that events *A* and *B* both occur can be found using the general multiplication rule

$$P(A \cap B) = P(A) \bullet P(B \mid A),$$

where $P(B \mid A)$ is the conditional probability that event *B* occurs given that event *A* has already occurred.

Conditional Probability and Independence
Two events *A* and *B* are **independent** if the occurrence of one event does not change the probability that the other event will happen. In other words, events *A* and *B* are independent if $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.

The Multiplication Rule for Independent Events
If *A* and *B* are independent events, then the probability that *A* and *B* both occur is

$$P(A \cap B) = P(A) \bullet P(B)$$